10.4 - 2nd

# Math 60 10.4 Graphing Quadratic Functions Using Transformations

Objectives 1) Graph quadratic functions (parabolas opening up or down) using algebraic structures

last lesson
- $f(x) = x^2 + k$ — vertical shift $k$ units
- $f(x) = (x-h)^2$ — horizontal shift $h$ units

this lesson
- $f(x) = ax^2$ — stretch/compress or upside-down
- $f(x) = a(x-h)^2 + k$ — combination of shifts and stretch/compress

2) Graph quadratic function given in the form

$$f(x) = ax^2 + bx + c$$

by using CTS (without solve) to rewrite as

$$f(x) = a(x-h)^2 + k^2$$

Math 60 10.4 - 2nd

① $f(x) = -x^2$

| x | y |
|---|---|
| 0 | 0 |
| 1 | -1 |
| 2 | -4 |
| 3 | -9 |
| 4 | -16 |
| -1 | -1 |
| -2 | -4 |
| -3 | -9 |
| -4 | -16 |

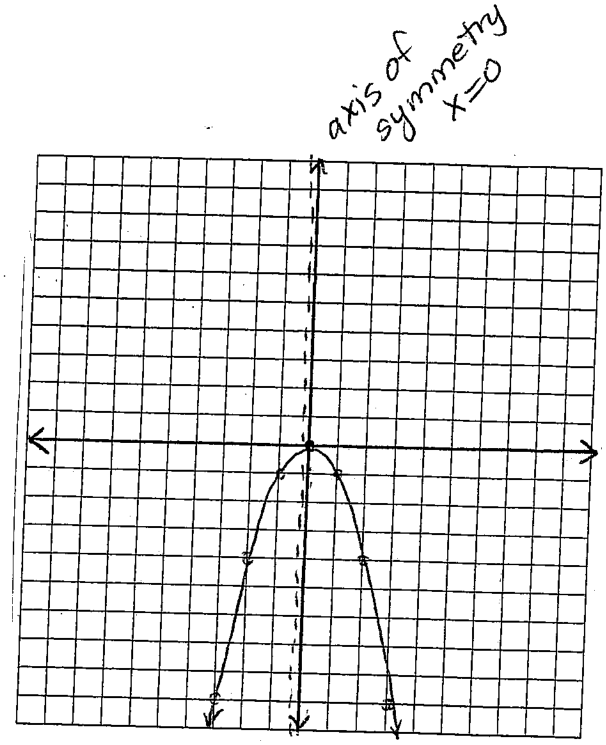
$-(1)^2 = -1$

$-(2)^2 = -4$

$-(-1)^2 = -1$

$-(-2)^2 = -4$

$-(-3)^2 = -9$




$f(x) = -x^2$

or

$f(x) = ax^2$ when $a<0$

means the parabola opens downward

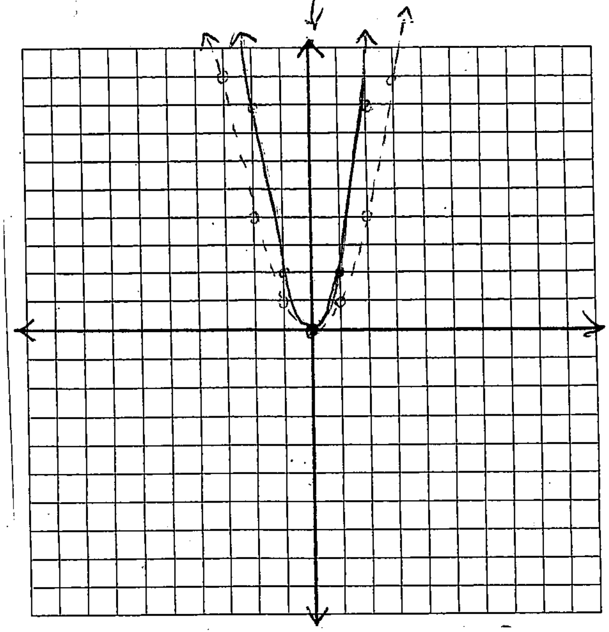
Math 60 10.4 - 2nd

Time? skip

(2) $f(x) = 2x^2$

| x | y | |
|---|---|---|
| 0 | 0 | $2(0)^2 = 0$ |
| 1 | 2 | $2(1)^2 = 2$ |
| 2 | 8 | $2(2)^2 = 8$ |
| 3 | 18 } not | $2(3)^2 = 18$ |
| 4 | 32 } helpful | |
| -1 | 2 | $2(-1)^2 = 2$ |
| -2 | 8 | |
| -3 | 18 } not | |
| -4 | 32 } helpful | |

axis of symmetry x=0



This parabola is narrower than the basic parabola $y=x^2$. (shown as dotted graph)

(3) $f(x) = \frac{1}{2}x^2$

| x | y |
|---|---|
| 0 | 0 |
| 1 | 1/2 |
| 2 | 2 |
| 3 | 9/2 |
| 4 | 8 |
| -1 | 1/2 } repeat |
| -2 | 2 |
| -3 | 9/2 |
| -4 | 8 |

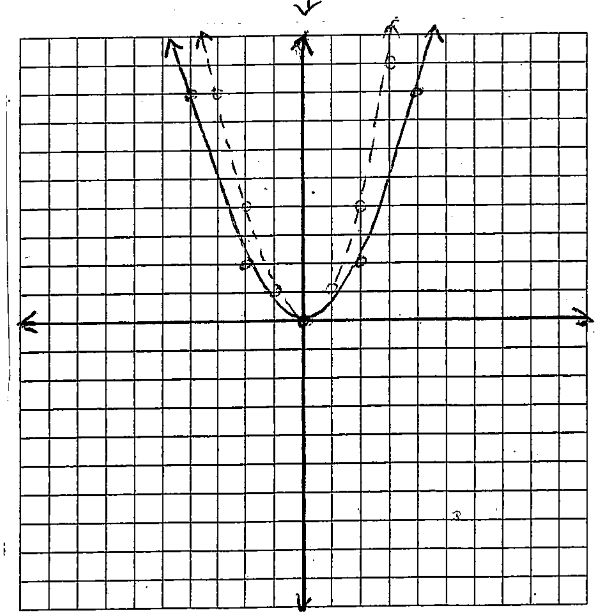
$\frac{1}{2}(2)^2 = 2$

$\frac{1}{2}(3)^2 = \frac{9}{2}$

$\frac{1}{2}(4)^2 = 8$

$\frac{1}{2}(-1)^2 = \frac{1}{2}$

axis of symmetry x=0



This parabola is wider than the standard parabola $y=x^2$

(shown as dotted graph)

Math 60   10.4 - 2nd

$f(x) = ax^2$

if $a > 1$   parabola opens up $(a > 0)$ and is narrower than the basic parabola $y = x^2$

if $0 < a < 1$   parabola opens up $(a > 0)$ and is wider than the basic parabola $y = x^2$

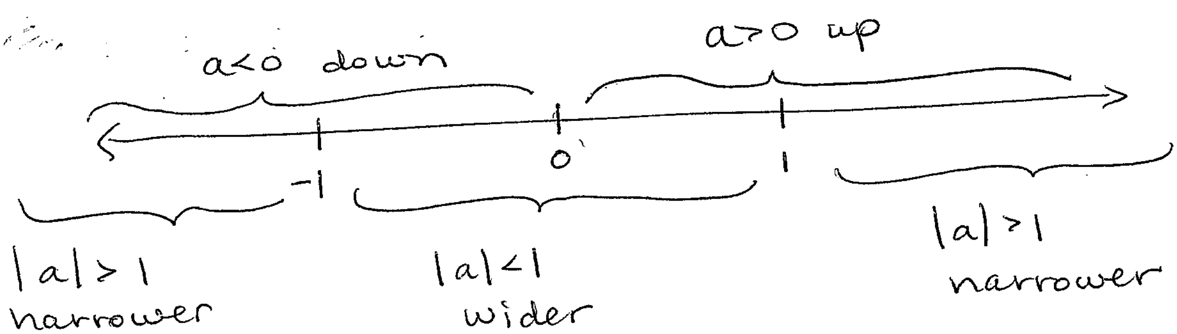
if $a < -1$   parabola opens down $(a < 0)$ and is narrower than the basic parabola $y = x^2$

if $-1 < a < 0$   parabola opens down $(a < 0)$ and is wider than the basic parabola $y = x^2$

if $a = 1$   it's the basic parabola

if $a = 1$   it's the basic shape, but opening downward.

On a number line:



Math 60 10.4 - 2nd

Time? skip Without graphing, identify

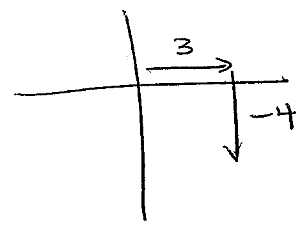
a) coordinates of vertex

b) direction parabola opens

c) whether parabola is standard, wider, or narrower.

d) equation of the axis of symmetry

(4) $f(x) = \underbrace{-2(x-3)^2}_{\text{shift right } 3} \underbrace{-4}_{\text{shift down } 4}$



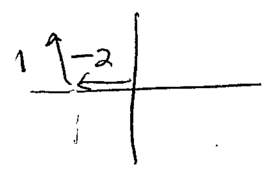

a) $\boxed{\text{Vertex } (3,-4)}$

b) $a=-2$ is negative $\boxed{\text{opens down}}$

c) $a=-2$ $\quad |a|>1$ $\boxed{\text{narrower}}$

d) $\boxed{x=3}$ axis of symmetry

(5) $f(x) = \underbrace{\frac{3}{4}(x+2)^2}_{\text{shift left } 2} \underbrace{+1}_{\text{shift up } 1}$




a) $\boxed{\text{Vertex } (-2,1)}$

b) $a=\frac{3}{4}$ is positive $\boxed{\text{opens up}}$

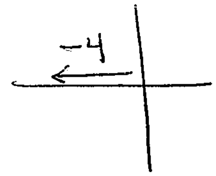
c) $a=\frac{3}{4}$ $\quad |a|<1$ $\boxed{\text{wider}}$

d) $\boxed{x=-2}$ axis of symmetry

Math 60 10.4 - 2nd

(6) $f(x) = -\frac{1}{3}(x+4)^2$

$\underbrace{-\tfrac{1}{3}(x+4)^2}_{\text{shift left 4}}$ $\underbrace{\phantom{+0}}_{\text{no up/down shift}}$




a) $\boxed{\text{vertex } (-4,0)}$

b) $a = -\frac{1}{3}$ is negative $\boxed{\text{opens down}}$

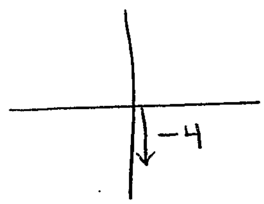
c) $a = -\frac{1}{3}$ $|a| < 1$ $\boxed{\text{wider}}$

d) $\boxed{x = -4}$ axis of symmetry

(7) $f(x) = -\frac{2}{3}x^2 - 5$

$x^2 = (x-0)^2$ no left/right shif

$\underbrace{-5}$ shift down 4




a) $\boxed{\text{vertex } (0,-4)}$

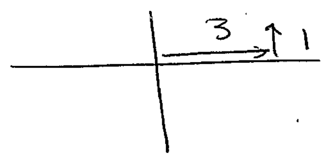
b) $a = -\frac{2}{3}$ is negative $\boxed{\text{opens down}}$

c) $a = -\frac{2}{3}$ $\left|\frac{2}{3}\right| < 1$ $\boxed{\text{wider}}$

d) $\boxed{x=0}$ axis of symmetry

(8) $f(x) = 4(x-3)^2 + 1$

$\underbrace{(x-3)^2}_{\text{shift right 3}}$ $\underbrace{+1}_{\text{shift up 1}}$




a) $\boxed{\text{Vertex } (3,1)}$

b) $a=4$ is positive $\boxed{\text{opens up}}$

c) $a=4$ $|a|>1$ $\boxed{\text{narrower}}$

d) $\boxed{x=3}$ axis of symmetry

Time? skip Math 60 10.4 - 2nd

(9) Write $f(x) = x^2 - 4x + 1$ in the form $f(x) = a(x-h)^2 + k$

Step 1: Identify all terms containing x. This must become $(x-h)^2$ when we are done.

$$f(x) = \underbrace{x^2 - 4x}_{\substack{(x-h)^2 \\ \text{eventually}}} \quad \underbrace{+1}_{\substack{\text{leftover} \\ \text{stuff.}}}$$

step 1.5: Notice it's plain $x^2$, no coefficient. (whew! dodged a bullet!)

step 2: Use the x-term coefficient to complete the square.

$\# = \frac{-4}{2} = -2$ ← use this number for factor

$\#^2 = (-2)^2 = 4$ ← add (and subtract) this number

$$f(x) = x^2 - 4x \underbrace{\underbrace{+4}_{\substack{\text{add} \\ \text{our} \\ \text{number}}} \quad \underbrace{-4}_{\substack{\text{subtract} \\ \text{our} \\ \text{number}}}}_{\text{net effect 0!}} \quad \underbrace{+1}_{\substack{\text{leftover} \\ \text{stuff}}}$$

$$f(x) = \underbrace{(x^2 - 4x + 4)}_{\substack{\text{this part} \\ \text{becomes} \\ \text{the perfect} \\ \text{square}}} + \underbrace{(-4+1)}_{\substack{\text{this part} \\ \text{becomes} \\ \text{the} \\ \text{vertical shift} \\ k}}$$

$$\boxed{f(x) = (x-2)^2 - 3}$$

Math 60 10.4 - 2nd

(10) Write $f(x) = -3x^2 - 12x + 1$ in the form $f(x) = a(x-h)^2 + k$

step 1: Identify all terms containing x.

$$f(x) = \underbrace{-3x^2 - 12x}_{x \text{ terms}} \quad \underbrace{+1}_{\text{stuff}}$$

step 2: Notice that $x^2$ has a coefficient that's not +1.
Bad news! Can't use CTS until we factor it out.

$$f(x) = \underbrace{-3(x^2 + 6x \qquad )}_{\text{factor coefficient } -3 \text{ from the } x\text{-terms only}} \quad \underbrace{+1}_{\text{leftover stuff.}}$$

step 3: Use the x-term coefficient to find numbers used to complete the square.

$\# = \frac{6}{2} = 3$ ← use this number for factor

$\#^2 = 3^2 = 9$ ← use this number to add inside ( ).

$$f(x) = -3(x^2 + 6x + 9) + 27 + 1$$

add 27 to un-do

add inside ( ) to get perfect square trinomial.
But mentally distribute:
$-3x^2 - 18x \underbrace{-27}$

we actually subtracted 27 to our function!

CAUTION
* when we add for CTS, we must have net effect 0

$$\boxed{f(x) = -3(x+3)^2 + 28}$$

Math 60 10.4 - 2nd

(11) Write $f(x) = \frac{1}{2}x^2 + 2x - 1$ in the form $f(x) = a(x-h)^2 + k$

Step 1: x-terms $\quad f(x) = \frac{1}{2}x^2 + 2x \quad -1$

Step 2: coef $\frac{1}{2}$! $\quad f(x) = \frac{1}{2}(x^2 + 4x \quad ) - 1$ $\quad$ factor out ½ means ÷ ½.

Step 3: CTS

$\# = \frac{4}{2} = 2$ ← # for factor

$\#^2 = 2^2 = 4$ ← $\#^2$ for adding

$f(x) = \frac{1}{2}(x^2 + 4x + 4) - 2 \quad -1$

mentally distribute

$\frac{1}{2}x^2 + 2x + 2$

⇓

We actually added 2. ⇒ subtract 2 for net effect 0.

leftover stuff from original question

$\boxed{f(x) = \frac{1}{2}(x+2)^2 - 3}$